Calculating Gravitational Wave Signatures from Black Hole Binary Coalescence

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Most of the information we have about the Universe has come to us in the form of . . .

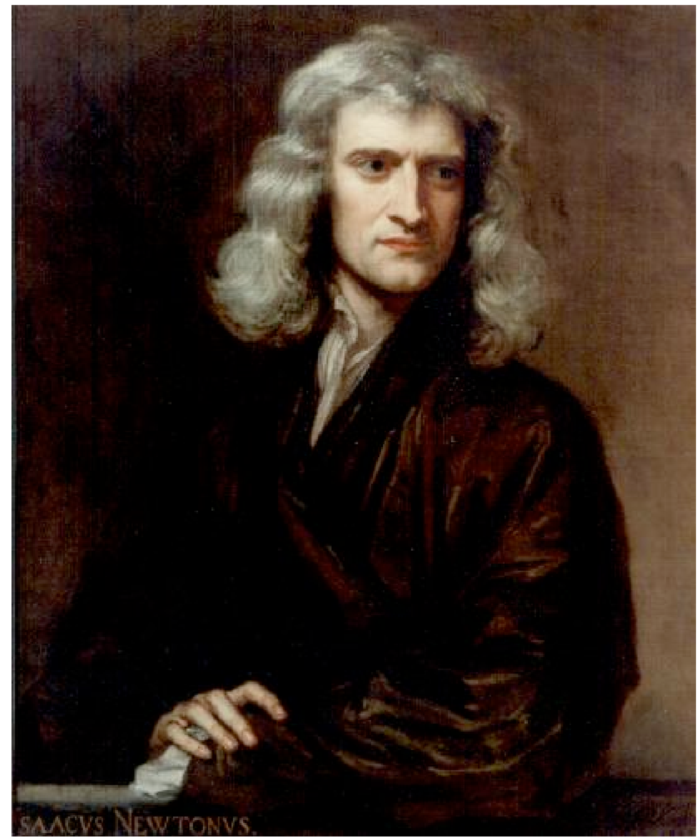
- Electromagnetic radiation
  - Visible light: naked eye observations,optical telescopes
  - Full electromagnetic spectrum: radio, IR, UV, visible, X-rays, Gamma-rays

- Particle and nuclear astrophysics, neutrinos, cosmic rays…

These cosmic messengers provide a wealth of information, making astronomy one of the crowning glories of 20th century science.

The gravitational force dominates the dynamics of the Universe . . .

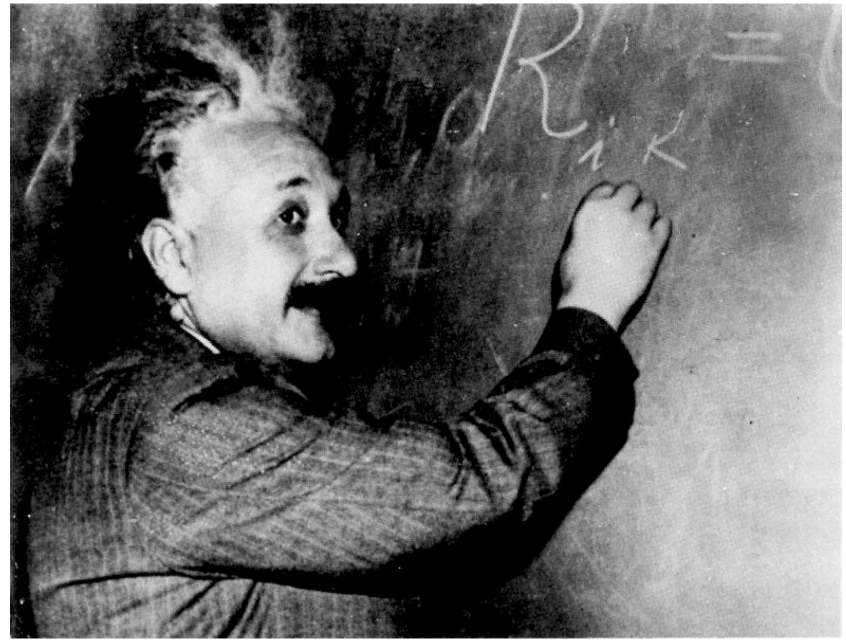
- Gravitational field – action at a distance
- Law of Universal Gravitation (1687)
- Fruitful legacy . . .
  - Solar system dynamics
  - Discovery of new planets, both solar and extra-solar
  - Motions of stars within galaxies
  - Motions of galaxies within clusters . . .



Isaac Newton (1642-1727)

The gravitational force dominates the dynamics of the Universe . . .

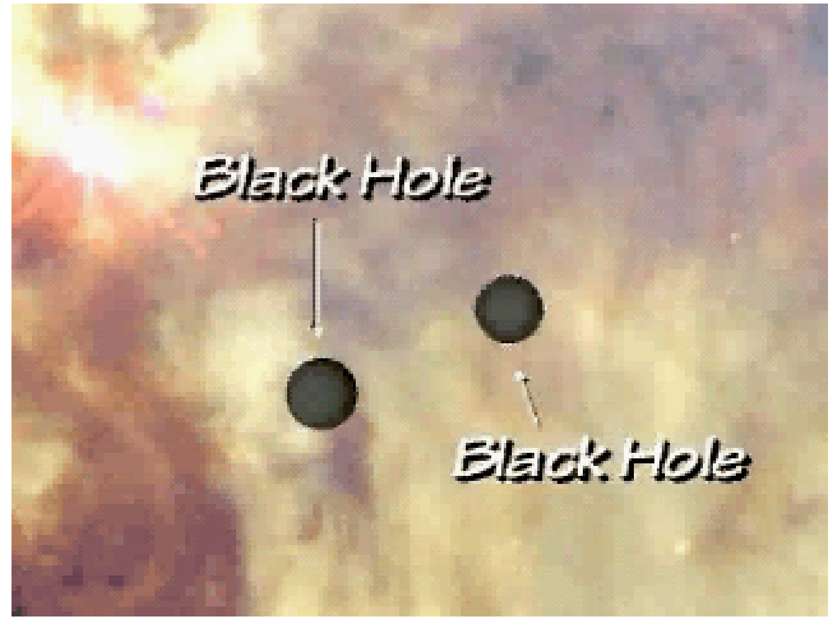
- Special relativity (1905)
  - Space + time → spacetime
  - Speed of light is constant
- General Relativity (1916)
  - Spacetime is dynamic
  - Spacetime curvature replaces concept of gravitational field
    - mass-energy causes spacetime to curve
    - particles & light follow paths in curved spacetime
    - Gravitational waves



Albert Einstein (1879 – 1955)

A Different Type of Astronomical Messenger
Gravitational Waves . . .

- ripples in spacetime curvature
- travel at velocity *v* = *c*
- generated by matter distributions w/ time-changing quadrupole moments → carry info about bulk motion of sources
- transverse → act normal to propagation direction
- 2 polarization states, h_+ and h_x
- interact weakly with matter → carry info about deep, hidden regions in the universe



- Hulse-Taylor binary pulsar PSR 1913+16
  - Orbital period decay agrees with GR to within the obs errors of < 1%
  - Nobel Prize 1993

Amplitudes of Gravitational Wave Sources . . .

- **Characteristic amplitude**

$$h \sim \frac{G}{c^4}\frac{\ddot{Q}}{r} \sim \frac{R_{Sch}}{r}\frac{v^2}{c^2}$$

  - **r = distance to source**
  - **$R_{Sch} = 2GM/c^2$**
  - **Q = (trace-free) quadrupole moment of source**
  - **v = characteristic nonspherical velocity in source**

→ **Strongest sources have large masses moving with velocities $v \sim c$**

Estimate upper limits:

- **1.4 M_{Sun} NS at**
  - **r = 15 kpc, $h \sim 10^{-17}$**
  - **r = 15 Mpc, $h \sim 10^{-20}$**
  - **r = 200 Mpc, $h \sim 10^{-21}$**
  - **r = 3000 Mpc, $h \sim 10^{-22}$**
- **2.5 x 10^6 M_{sun} MBH at**
  - **r = 3000 Mpc, $h \sim 10^{-16}$**

Estimating Gravitational Wave frequencies . . .

- **Natural frequency**

$$f_o \sim \left(\frac{\bar{\rho}G}{4\pi}\right)^{1/2} \sim \frac{\sqrt{3}}{4\pi}\left(\frac{GM}{R^3}\right)^{1/2}$$

- **1.4 M_{Sun} NS, R = 10 km**
  $f_o \sim$ 2 kHz
- **10 M_{Sun} BH**
  $f_o \sim$ 1 kHz
- **2.5 x 10^6 M_{Sun} MBH**
  $f_o \sim$ 4 mHz

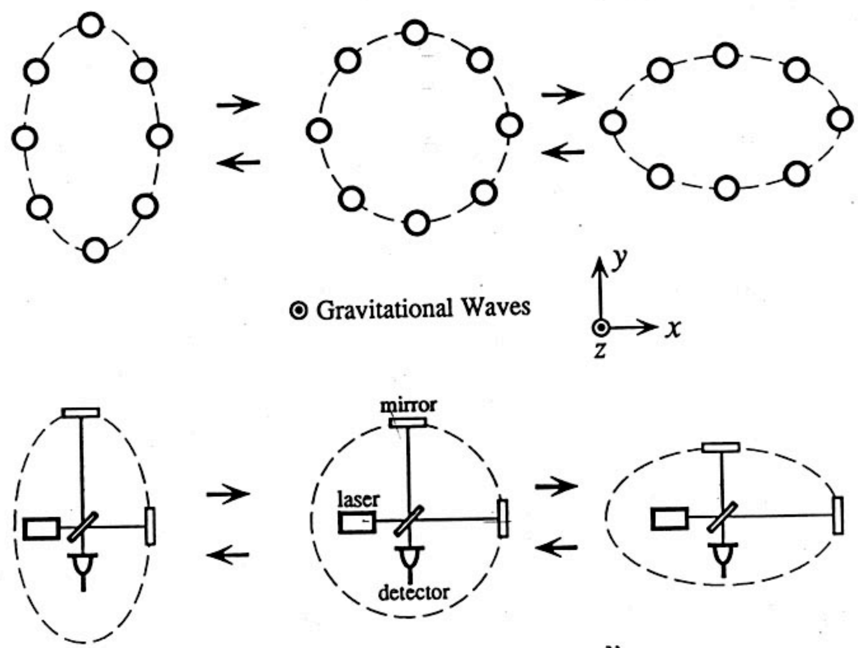
- **Binary orbital frequency**

$$f_{\mathrm{GW}} = 2 f_{\mathrm{orb}} = \frac{1}{\pi}\left(\frac{GM}{a^3}\right)^{1/2}$$

  - **$M = M_1 + M_2$, $M_1 = M_2$**
  - **a = separation**
- **NS/NS, a = 10 R**
  $f_{GW} \sim$ 200 Hz
- **BH/BH, a = 10 M**
  $f_{GW} \sim$ 100 Hz
- **MBH/MBH, a = 10 M**
  $f_{GW} \sim 4 \times 10^{-4}$ Hz

Detecting gravitational waves. . .

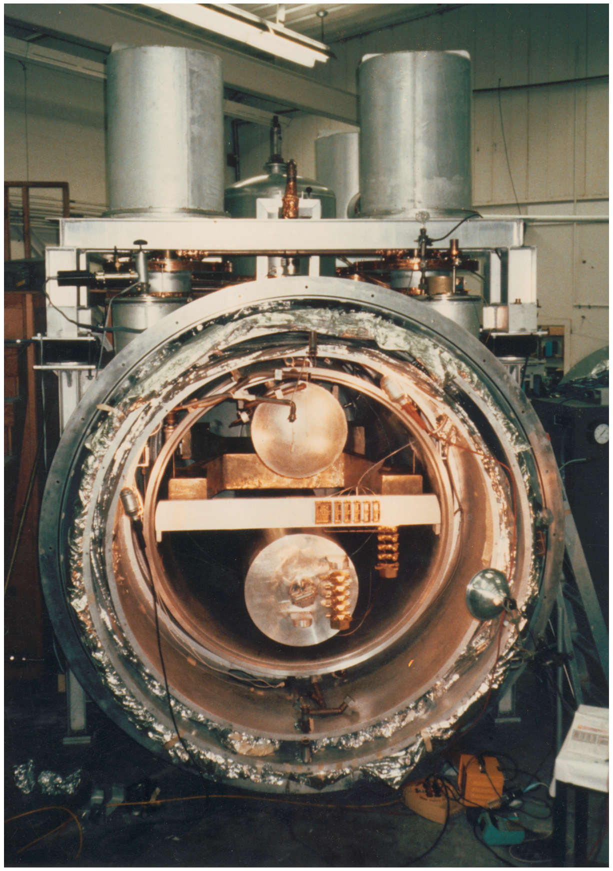
- Resonant mass detectors, laser interferometers
- Detector of length scale *L*
- A passing gravitational wave causes distortion of detector that produces a strain amplitude *h(t) = _L/L*
- Source waveforms scale as *h(t) ~ 1/r*



(graphic courtesy of B. Barish, LIGO-Caltech)

Resonant Mass Detectors…..

- Pioneered by Weber
- Measure distortions of large "bar"
- Narrow band
- Spherical detectors
- International Gravitational Event Collaboration (IGEC)
  - Rome, Legnaro, Perth, LSU
  - http://igec.lnl.infn.it



The Allegro detector at LSU
principal sensitivity ~ 920 Hz
(image courtesy of W. Hamilton)

Ground-based interferometers . . .

- detect high frequency GW

$$10\ \text{Hz} \le f_{\text{GW}} \le 10^4\ \text{Hz}$$

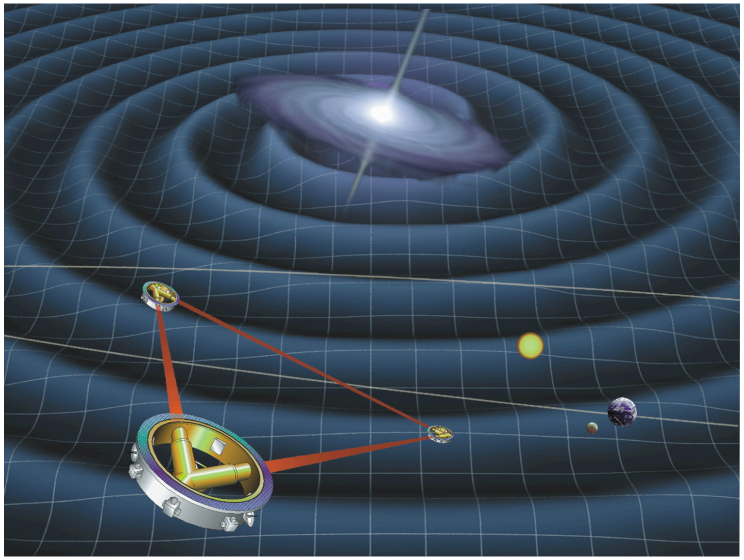
- broad band
- kilometer-scale arms
- Current projects:
  - LIGO: Hanford, WA, and Livingston, LA; $L = 4$ km
  - VIRGO: France/Italy, near Pisa; $L = 3$ km
  - GEO600: Germany/Britain, Hanover; $L = 600$ m
- Typical sources: NS/NS, BH/BH, stellar collapse, LMXBs, . . .

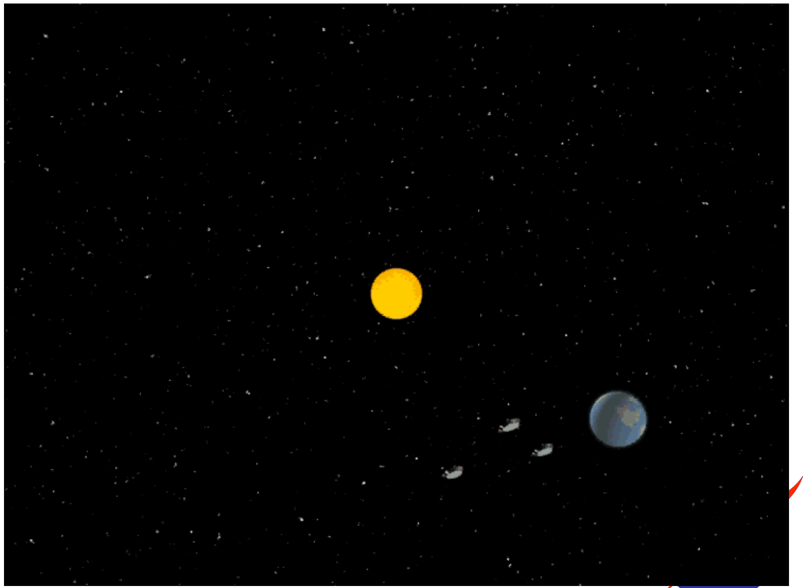
LISA: Laser Interferometric Space Antenna

- NASA/ESA collaboration
- detect low frequency GW

$$10^{-4}\ \mathrm{Hz} \leq f_{\mathrm{GW}} \leq 1\mathrm{Hz}$$

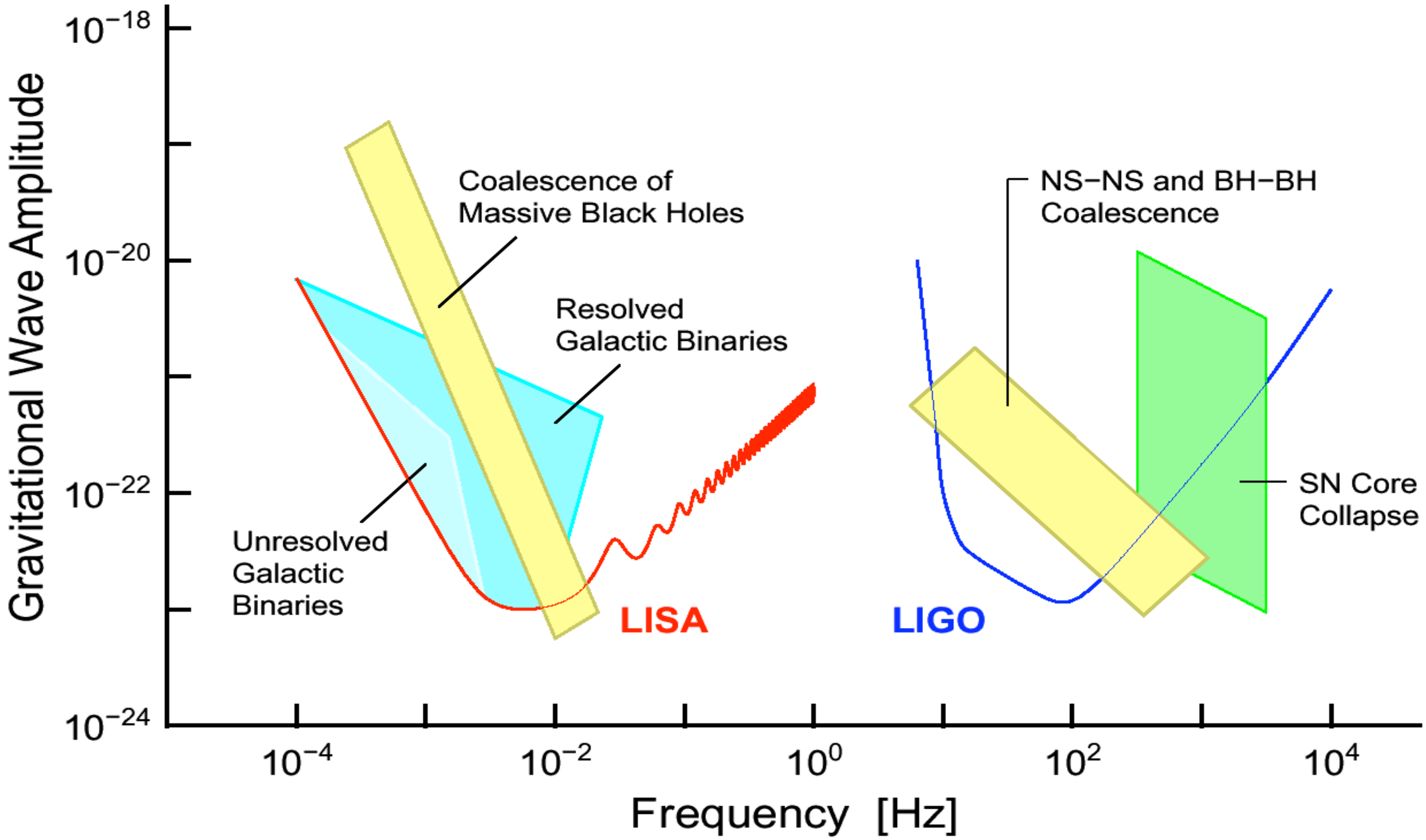
- 3 spacecraft
  - equilateral triangle
  - orbits Sun at 1 AU
  - 20° behind Earth in its orbit
- arm length $L = 5 \times 10^6$ km
- optical transponders receive and re-transmit phase locked light
- launch ~ 2013
- Typical sources: MBH/MBH, Galactic binaries, NS/MBH, BH/MBH





LISA / LIGO Relationship

- Complementary observations, different frequency bands
- Different astrophysical sources



The rich variety of sources implies GWs will tell us much about the universe….

- **Collapses**
- **Oscillations and deformations**
- **Binaries**
- **Gravitational captures**
- **Backgrounds and bursts**
- **Serendipity…**

* ste

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* compact stellar remnants (WD, NS, BH)

* stochastic backgrounds of confusion-limited sources

* cosmological GW

* unexpected sources…
  * dark matter?
  * dark energy?
  * ??

*Super

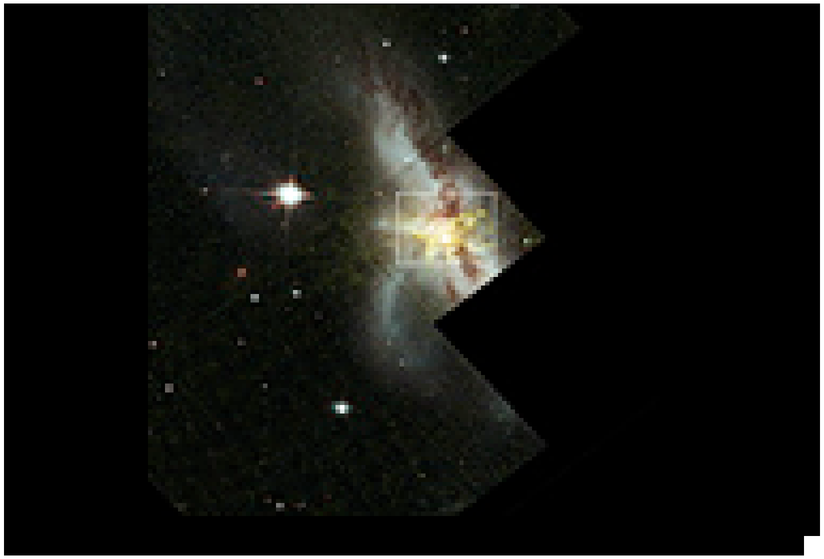
(M

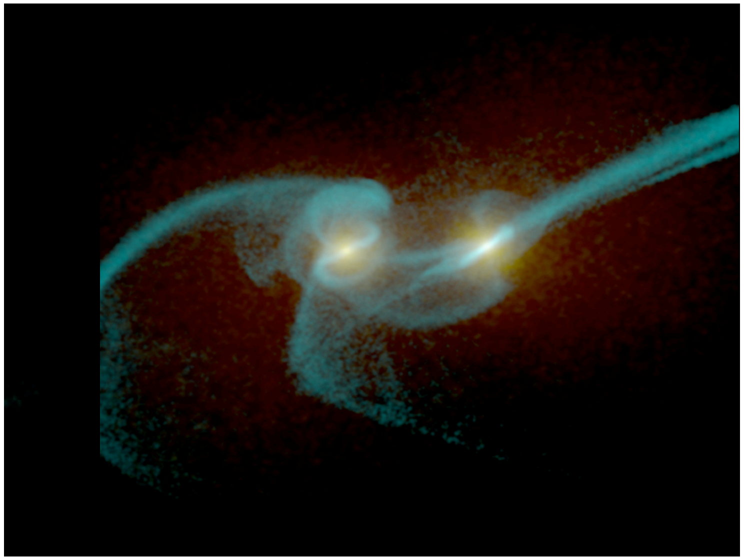
Focus here on the final coalescence of binary black holes…

and what we can learn about astrophysics and the cosmos by observing the gravitational waves they emit….

Coalescing supermassive BH binaries….

- Supermassive BHs lurk at the centers of most, if not all, galaxies
- Masses $M \geq 10^5\ M_{Sun}$
- Chandra X-ray observatory found the first known system of 2 SMBH starting to merge in the galaxy NGC 6240
  - distance ~ 120 Mpc → close!
  - BHs will merge in ~ few x 10^8 years
- Most galaxies are formed from the merger of 2 progenitor galaxies → merger of SMBHs
- LISA could observe roughly several per year, out to redshifts z > 10





Coalescing intermediate mass or seed BH binaries…

- Black holes having masses $M \sim$ few x $10^2\ M_{Sun} - 10^4\ M_{Sun}$
- Predicted in hierarchical structure formation theories:
  - **galaxies form from successive mergers of protogalactic fragments**
  - **SMBHs at the centers of galaxies form from successive mergers of smaller "seed" BHs at the centers of these fragments**
- IMBH also can form
  - **from the collapse of massive Pop III stars that form BHs**
  - **in stellar clusters from successive mergers of lower mass BHs**
- LISA will be able to detect these systems out to redshifts $z \sim 7 - 30$

  → will give an unprecedented view of the merger history of galaxies
- Ground-based detectors will see the final coalescence of systems with masses ~ few x $10^2\ M_{Sun}$

Coalescing stellar black hole binaries…

- Black holes having masses $M \sim$ few x 10 - 10^2 M_{Sun}
- Stellar BHs are formed as the end product of the core collapse of massive stars
  - **if mass of remnant core ~ 2 M_{Sun} or larger → BH will form**
  - **BH may also form from fallback of gas onto NS, causing collapse**
- Excellent source for ground-based interferometers….

Final Coalescence of BH binary…..

- Dominant energy loss mechanism is GW emission
- Coalescence time for binary of total mass *M* and separation *a* (equal point masses, circular orbits):

$$t_{\rm GR} = \frac{5}{64}\frac{c^5}{G^3}\frac{a}{M^3}$$

- For binary of total mass $M = 2 \times 10^6$ M_{Sun} to coalesce within $t_H \sim 10^{10}$ years

  → separation $a < a_{max} \sim 2.53 \times 10^4\, M \sim 2.4 \times 10^{-3}$ pc
- GW detectors will observe the end stages of this coalescence, typically $\sim 10^3$ orbits
  - LISA: will observe massive BH binaries for ~ 1 year
  - Ground-based detectors: will observe stellar BH binaries for ~ 1 minute

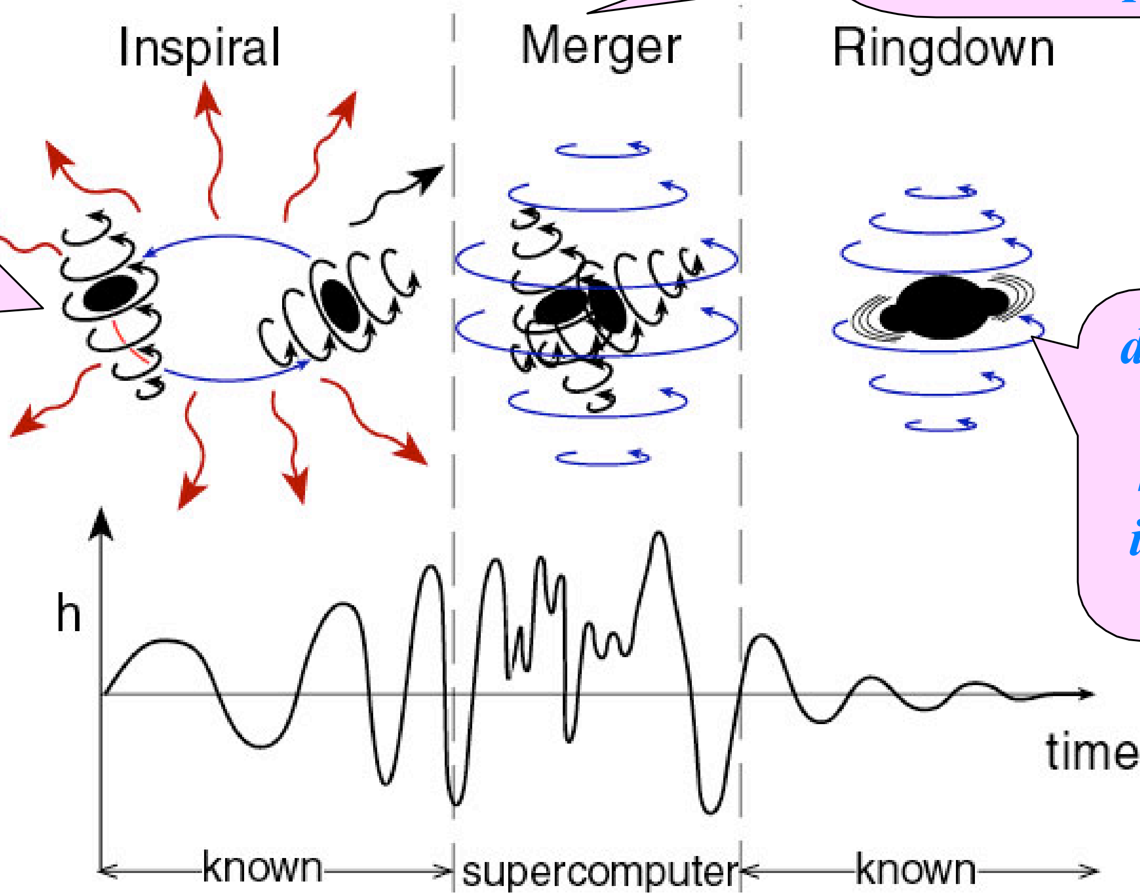
Final coalescence proceeds in 3 stages . . .

- GW produced in all three phases of this evolution . . .
- Waveforms and dynamics scale with BH masses and spins

  → source modeling applicable to stellar BHs, IMBHs & SMBHs….

strong-field spacetime dynamics, spin flips and couplings…

measure masses and spins of binary BHs



detect normal modes of ringdown to identify final Kerr BH

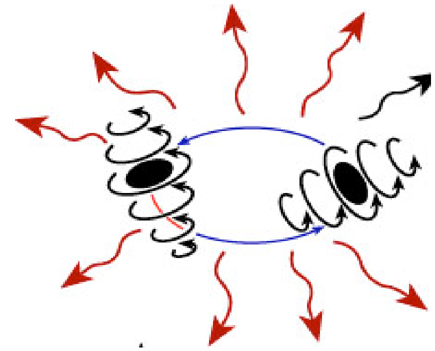
(graphic courtesy of Kip Thorne)

Inspiral stage…

- Slow, quasi-adiabatic inspiral driven by GW emission

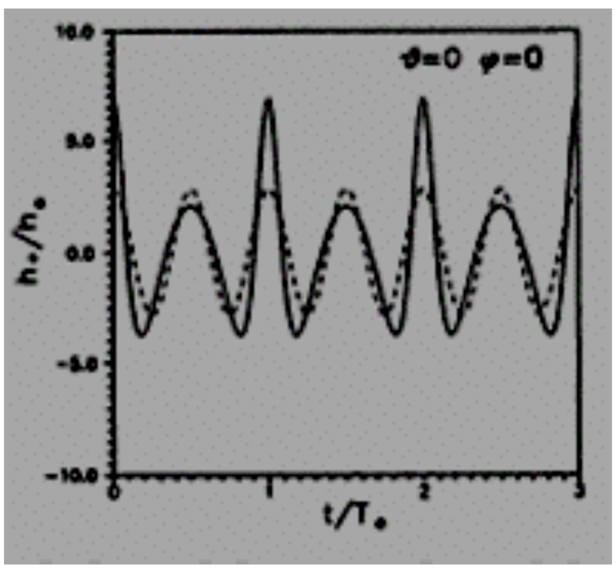
"chirp" waveform: sinusoid increasing in amplitude & frequency as the BHs get closer together

$$f_{\text{binary}} = \frac{2 f_{\text{orbital}}}{(1+\text{z})} = \frac{1}{\partial}\frac{1}{(1+z)}\left(\frac{GM}{a^3}\right)^{1/2}$$

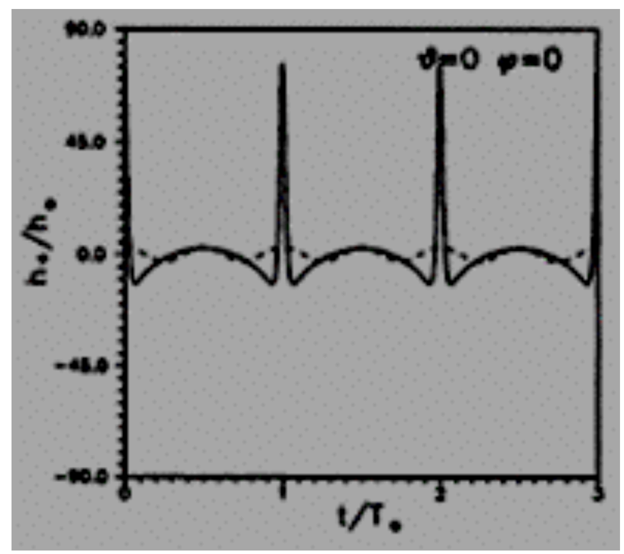


– **Eccentricity can alter waveform shape significantly (Pierro, et al.)**

e = 0.274 (PSR 1534+12)

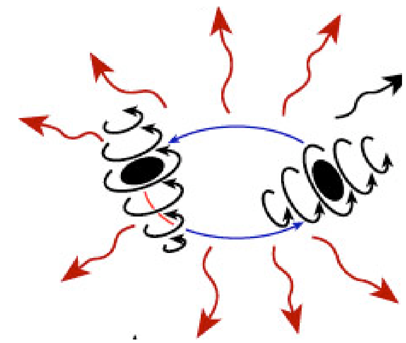


e = 0.617 (PSR 1913+16)



– **Also, precession effects due to BH spin (Vecchio, Kalogera….)**

Inspiral stage:

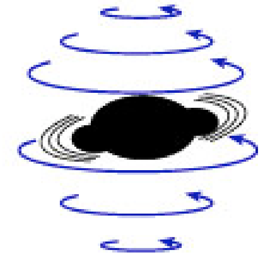


- MBH observable by LISA for ~ months to years
- Use waveforms as templates for data analysis

  by matched filtering
- If observe a sufficient number of cycles of inspiral waveform (for LISA, ~ few months or longer) within the detector's frequency band, can measure redshifted masses (1+z)M:
  - Chirp mass $\mathbf{M_c = (M_1M_2)^{3/5}/[M_1 + M_2]^{1/5}}$
  - Reduced mass (less accurately) _ $\mathbf{= M_1M_2/[M_1 + M_2]}$
  - Also some information on spins…
- If know cosmology to ~ 10%, invert luminosity distance relation D_L (z) to get redshift z → M_{tot} (Hughes 2002)
- Typically, get (1+z)M_c to ~ 0.1% or better

  → M_c to ~ 15 – 30%

Ringdown…



- **Merger → rotating, highly distorted BH**
- **"Rings down" to a quiescent Kerr BH by emitting GW**
- **Ringdown waveforms are *exponentially damped sinusoids*, dominated by the strongest l = m = 2 quasinormal mode**

$$f_{\text{ring}} = \frac{f_{\text{QNR}}}{(1+z)} \cong \left[1 - 0.63(1-\hat{a})^{3/10}\right]\left\{\frac{10^6 \text{M}_{\text{SUN}}}{M}\right\}\left(\frac{3.2\text{x}10^{-3}\,\text{Hz}}{(1+z)}\right), \quad 0 \le \hat{a} \le 1$$

(Echeverria; Leaver)

- **Quality factor** $$Q \cong 2(1-\hat{a})^{-9/20} = \partial \tau_{\text{damp}} f_{\text{QNR}}$$

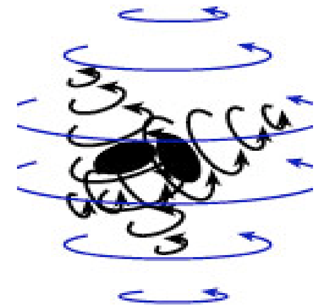
- **Ringdowns are "burst" waveforms**

For M = 2 x 10⁶ M$_{\text{Sun}}$, $\text{if } \hat{a} = 0.5, \quad f_{\text{ring}} \approx \frac{7.8\text{x}10^{-3}\,\text{Hz}}{(1+\text{z})}, \; \tau_{\text{damp}} \approx 6M \approx 1\text{hr}$

$$\text{For } \hat{a} = 0.98, \;\; \tau_{\text{damp}} \approx 29M \approx 4.8\text{hr}$$

- **Note: we observe redshifted damping timescale** $(1+z)\tau_{\text{damp}}$
- **identify mass and spin of final Kerr BH**

Focus on the merger stage…

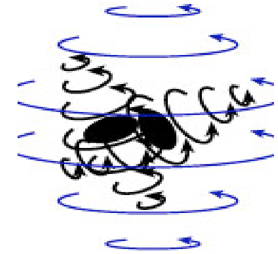


- Inspiral lasts until separation $a \sim 3R_{\mathbf{Schw}} = 6M$

$$\rightarrow \quad f_{\text{inspiral}} \leq 4 \text{ x } 10^{-3} \text{ Hz} \left(\frac{10^6 \text{M}_{\text{Sun}}}{(1+\text{z})\, M} \right)$$

- BHs leave quasi-static orbits and plunge together
- Expect ~ several cycles of gravitational radiation from merger
  - → **"burst" waveform, observable by LISA for ~ minutes – hours**
  - → **knowledge of merger waveform important to enhance detectability in ground-based GW observatories….**
- Merger can be phenomenologically rich
  - **effects of spin: spin-spin and spin-orbit couplings, spin flips**
  - **possible ejection of final BH for $M_1 \neq M_2$**
  - **test of General Relativity in the dynamical, nonlinear regime**
- Strong, highly nonlinear, dynamical gravitational fields
- Requires numerical solution of the full Einstein equations in 3 spatial dimensions + time…

Using numerical relativity to evolve BH mergers…



- Einstein equations: ***(a,b = 0,1,2,3 – spacetime indices)***

$$ {}^{(4)}G_{ab} \equiv {}^{(4)}R_{ab} - \frac{1}{2}{}^{(4)}R\, g_{ab} = 8\pi T_{ab}; $$

- "3+1" → split spacetime into 3-D spatial slices + time
- Kinematical conditions: freely specificable gauge choices
  - **lapse function _: measures proper time (_ dt) between slices**
  - **shift vector _^{i}: allows moving of spatial coordinates as you evolve from slice to slice**
- Metric becomes ***(i,j = 1,2,3 – spatial indices)***

$$ g_{ab} = \begin{pmatrix} -\alpha^2 + \beta_k \beta^k & \beta_i \\ \beta_j & \gamma_{ij} \end{pmatrix} $$

- Choice of _, _^{i} of critical to successful spacetime evolutions….

Einstein equations split into 2 sets….

- Constraint equations set conditions on spacelike slices
  - Hamiltonian constraint: $R + K^2 - K_{ij}K^{ij} = 16\pi\rho.$
  - Momentum constraints: $D_j K^j{}_i - D_i K = 8\pi j_i.$
  - Constraints also serve as initial value equations
- Evolution equations in ADM *(Arnowit, Deser, Misner)* form: 1st order in time, 2nd order in space
  - 3-metric: $\partial_t \gamma_{ij} = -2\alpha K_{ij} + D_i\beta_j + D_j\beta_i$
  - extrinsic curvature:

$$\partial_t K_{ij} = -D_i D_j \alpha + \alpha(R_{ij} - 2K_{ik}K^k{}_j + KK_{ij}) \\ -\alpha 8\pi(S_{ij} - \frac{1}{2}\gamma_{ij}(S - \rho)) \\ +\beta^k D_k K_{ij} + K_{ik}D_j\beta^k + K_{kj}D_i\beta^k.$$

Initial data for BH mergers….

- **Goal:** initial data for BHs representing the ***realistic astrophysical state*** of a binary that has spiralled in by the emission of gravitational radiation
- Need to solve 4 nonlinear constraint equations on initial slice
- Most techniques use a conformal decomposition *(York, et al…)*
  - Split basic variables into ***freely specifiable*** and ***constrained***
  - Choose the freely specifiable pieces
  - Constraints determine the remaining variables
    → guaranteed solution to initial value equations
- physical 3-metric _ij : $\gamma_{ij} = \psi^4 \tilde{\gamma}_{ij}$ _ is conformal factor
- split extrinsic curvature into trace and trace-free parts:

$$K^{ij} = A^{ij} + \frac{1}{3}\gamma^{ij} K$$

- Various approaches further decompose A^{ij}….

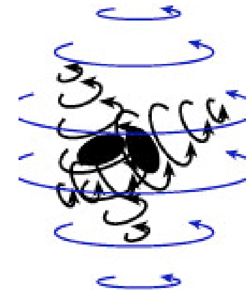
How to be sure the initial data is astrophysically valid?

- Initial data sets generally contain gravitational radiation due to the selection of free variables and the solution process
  - **these have largely been chosen to simplify the solution and decouple the constraint equations**
- Most current data sets for BH binaries near the last stable orbit contain (likely unphysical) gravitational radiation ~ few percent of the total system mass *(Pfeiffer, Cook, Teukolsky 2002)*
- ***This is comparable to the total GW energy expected for the merger process itself !***

→ Adopt a more physical approach….

- **use binary parameters determined by PN expansion near the end of the inspiral to inform the selection of free variables**
- **challenging: the metric and extrinsic curvature that emerge from the solution process can be changed significantly from the PN input**
- **recent work by Tichy, et al. shows promise….**

Evolving BH binary mergers…



- Need to evolve BH binary for ~ 3.5 orbits near the last stable orbit (LSO) at the end of the inspiral, through merger and ringdown…and extract the GW signature
- Orbital period near LSO is $P \sim 70M$ (possibly larger…)

  → ***need total evolution time of ~ 1000 M or more***
- *Challenges:*
  - **choice of formalism → critical for long term stability of evolution**
  - **how to represent the BHs on a computational grid**
  - **choice of gauge: slicing and conditions on spatial coordinates**
  - **multiple spatial and temporal scales → adaptive mesh refinement**
  - **boundary conditions**
  - **choice of numerical methods (finite differencing, spectral methods)**
  - **role of the constraint equations in evolution schemes**
  - **parallelism and efficiency of computer code…..**

Evolving BHs…effects of the choice of formalism

- Original ADM unstable for binary BH evolutions after ~ 13 *M*
  - **exponentially growing unstable modes**
  - **exact cause of instability is not yet known**
- Conformal ADM formalisms: much better stability properties
  - **BSSN: Baumgarte, Shapiro, Shibata, Nakamura**
  - **variations being developed to "tune" the system of equations and allow longer-lived evolutions**
  - **currently, can evolve a single BH for > several x 10^3 M (*Yo, et al.)***
  - **binary black hole evolutions now possible for ~ 100 M or longer *(Alcubierre, et al.)***
  - **note: longevity also depends on gauge choices….**
- Hyperbolic formulations: "mathematically desirable"
  - **fully first order set of equations**
  - **introduce relatively large set of auxiliary variables**
  - **stability of single BH runs > few x 10^3 M**
  - **area of active research….**

Evolving BHs…choice of formalism

- **BSSN or conformal ADM formalisms – have much improved stability properties though with a larger set of evolution equations:**

$$\partial_t \ddot{o} = -\tfrac{1}{6}\alpha\, K + \hat{\text{a}}^i \partial_i \ddot{o} + \tfrac{1}{6}\partial_i \hat{a}^i$$

$$\partial_t K = -\gamma^{ij} D_j D_i \alpha + \alpha(\tilde{A}_{ij}\tilde{A}^{ij} + \frac{1}{3}K^2) + 4\pi\alpha(\rho + S) + \beta^i \partial_i K$$

$$\partial_t \bar{\gamma}_{ij} = -2\alpha \tilde{A}_{ij} + \beta^k \partial_k \bar{\gamma}_{ij} + \bar{\gamma}_{ik}\partial_j \beta^k + \bar{\gamma}_{kj}\partial_i \beta^k - \frac{2}{3}\bar{\gamma}_{ij}\partial_k \beta^k.$$

$$\partial_t \bar{\Gamma}^i = -2\tilde{A}^{ij}\partial_j \alpha + 2\alpha\Big(\bar{\Gamma}^i_{jk}\tilde{A}^{kj} - \tfrac{2}{3}\bar{\gamma}^{ij}\partial_j K - 8\pi\bar{\gamma}^{ij}S_j + 6\tilde{A}^{ij}\partial_j \phi\Big)$$
$$+\beta^j \partial_j \bar{\Gamma}^i - \bar{\Gamma}^j \partial_j \beta^i + \tfrac{2}{3}\bar{\Gamma}^i \partial_j \beta^j + \tfrac{1}{3}\bar{\gamma}^{li}\beta^j_{,jl} + \bar{\gamma}^{lj}\beta^i_{,lj}.$$

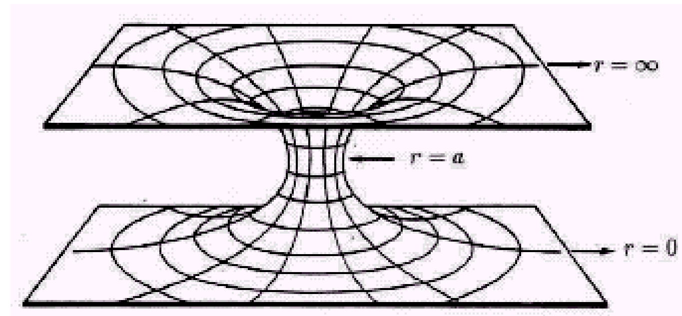
$$\partial_t \tilde{A}_{ij} = e^{-4\phi}\Big(-(D_i D_j \alpha)^{TF} + \alpha(R^{TF}_{ij} - 8\pi S^{TF}_{ij})\Big)$$
$$+\alpha(K\tilde{A}_{ij} - 2\tilde{A}_{il}\tilde{A}^l{}_j)$$
$$+\beta^k \partial_k \tilde{A}_{ij} + \tilde{A}_{ik}\partial_j \beta^k + \tilde{A}_{kj}\partial_i \beta^k - \tfrac{2}{3}\tilde{A}_{ij}\partial_k \beta^k.$$

- **plus additional constraint:**

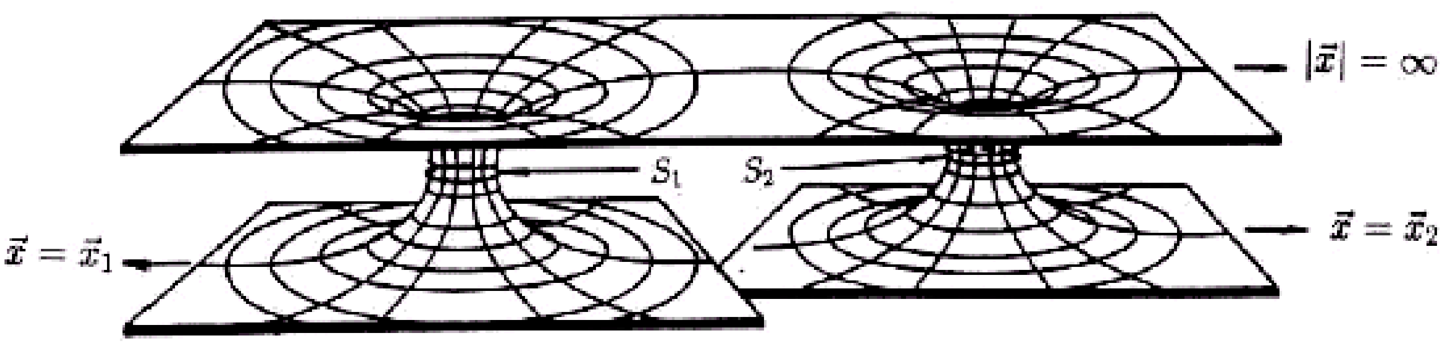
$$\bar{\Gamma}^i \equiv \bar{\gamma}^{jk}\bar{\Gamma}^i_{jk} = -\bar{\gamma}^{ij}{}_{,j}.$$

Evolving BHs…how to represent the BHs

- How to handle the singularity….
  - **horizon at r=2M**
  - **asymptotically flat as r → ∞**
  - **map within horizon as r → 0 to an inner asymptotically flat region**

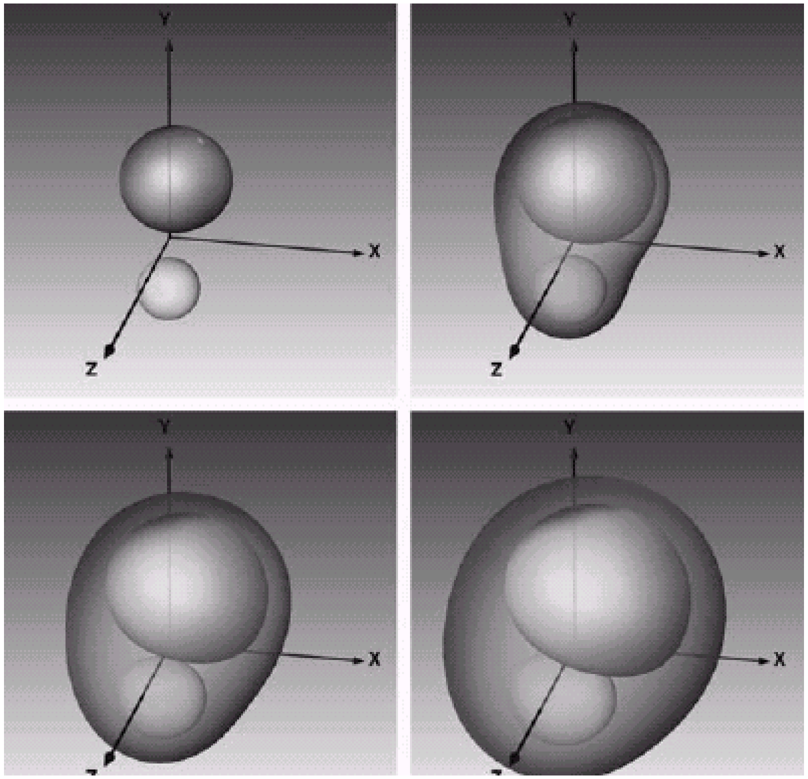


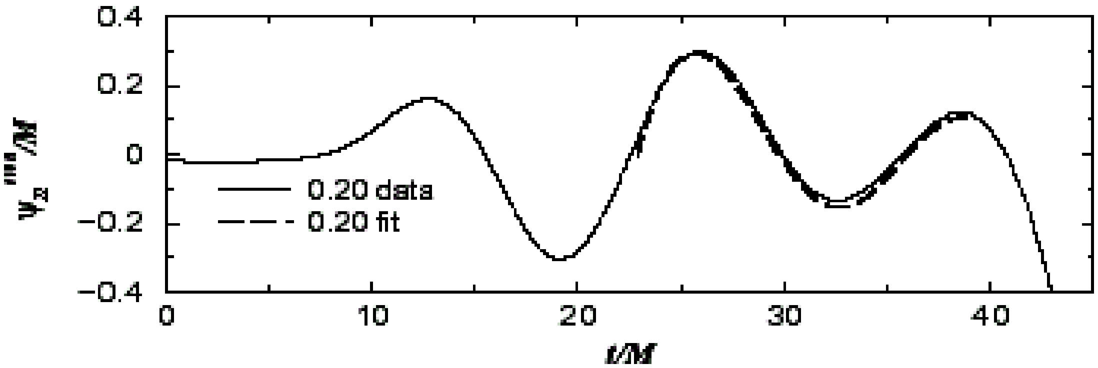
- "Puncture" method:
  - **conformal factor _ contains 1/r terms at the centers of the BHs**
  - **factor these out and treat them analytically → no singular terms for initial data or evolution**
  - **works for one or more BHs**



Grazing collision of BHs *(Alcubierre, et al.)*

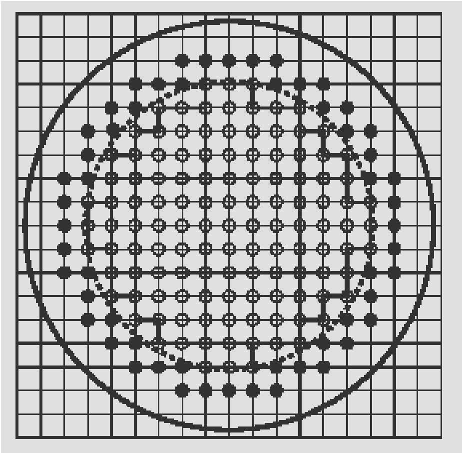
- Puncture evolution (_$^{i} = 0$)
- $M_1 = 1.5\ M_2$
- BHs have general spins, momenta
- Evolve for t = 35 M
- Formation of common apparent horizon
- Extract $l = m = 2$ GW mode



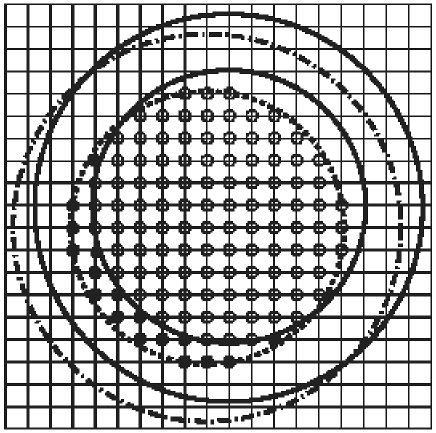


Evolving BHs…how to represent the BHs

- Excision: cut out a region containing singularity
  - *Event horizon*: boundary between the events which emit light rays to ∞ and those which do not
    → requires knowledge of entire spacetime
  - *Apparent horizon*: useful for evolutions
    - outermost 2-surface in spatial slice whose outgoing null geodesics have zero expansion
    - is always located within an event horizon
      → it is safe to excise w/in AH

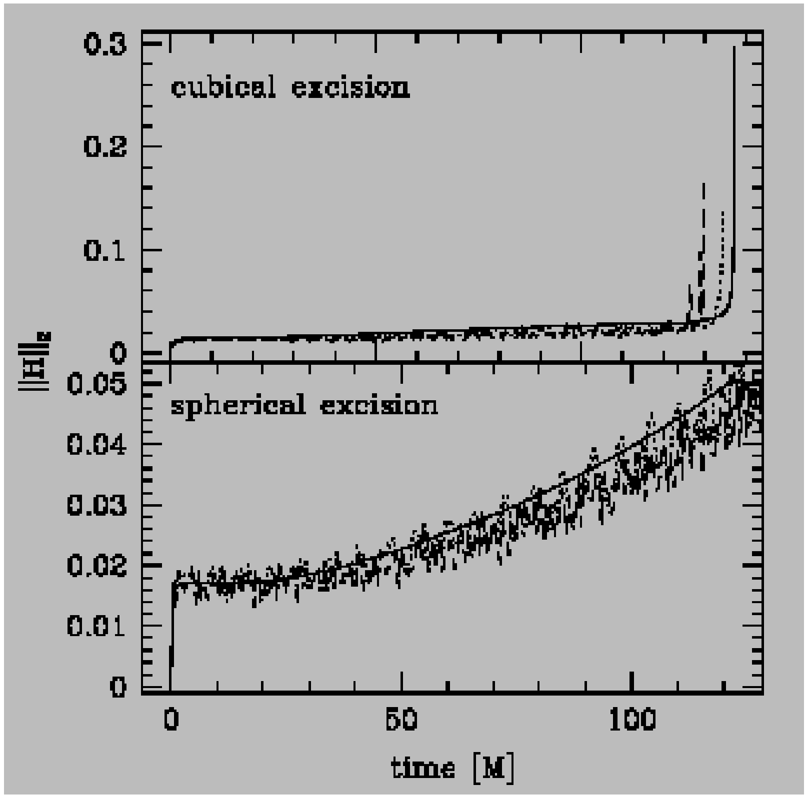


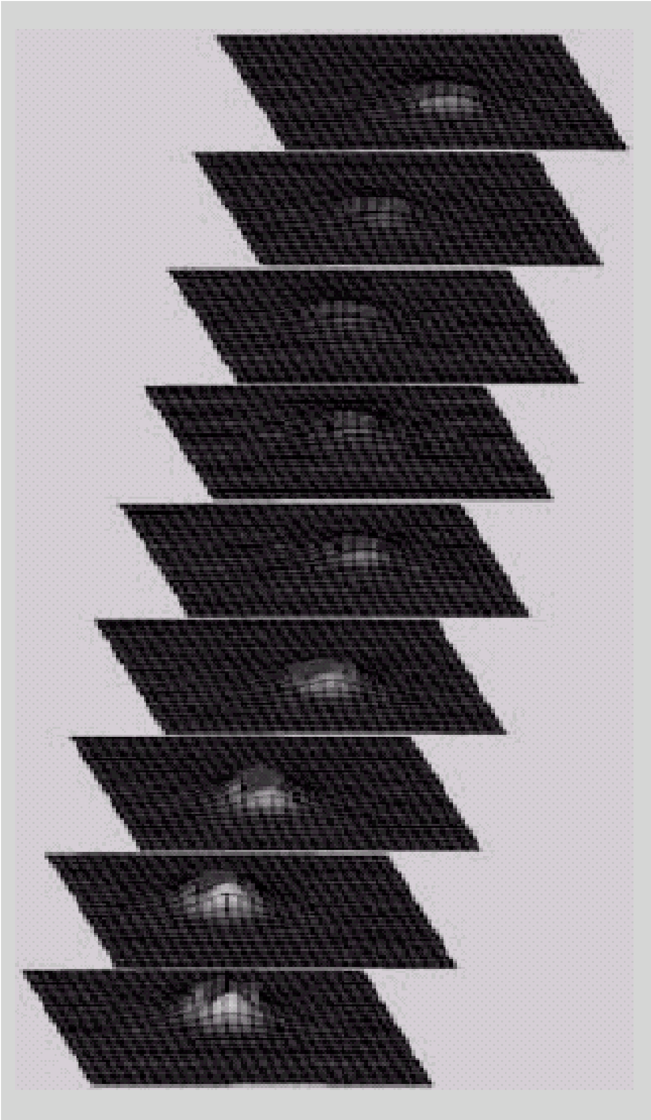
  - Need to set boundary conditions at excision
  - When an excised BH is moved, zones that were previously within excised region are now on the numerical grid
    → need methods to populate these points with data



Moving a BH with excision… *(Shoemaker, et al.)*

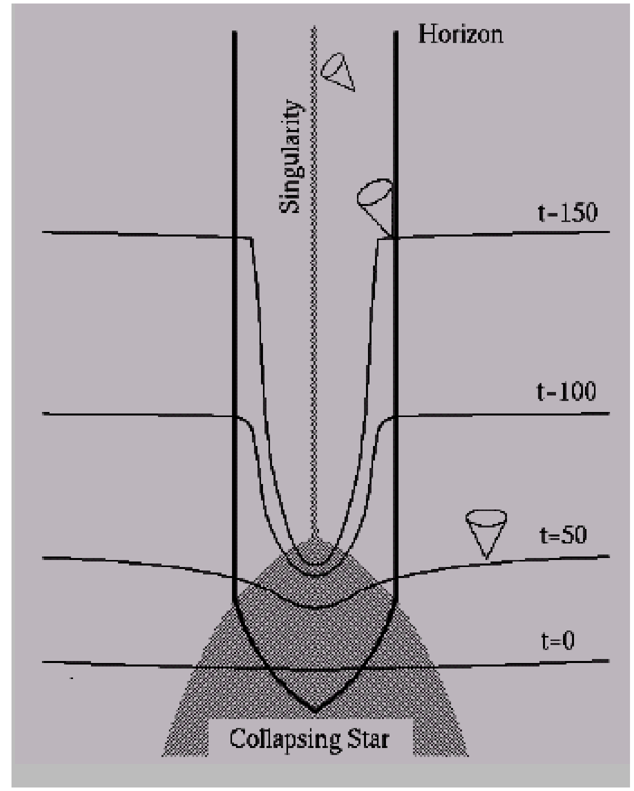
- Move excised BH using coordinate transformation of stationary BH enabled by shift _^i
- Cubical, spherical excision regions
- Stable for time t ~ 100 *M*





Evolving BHs…choice of lapse and shift

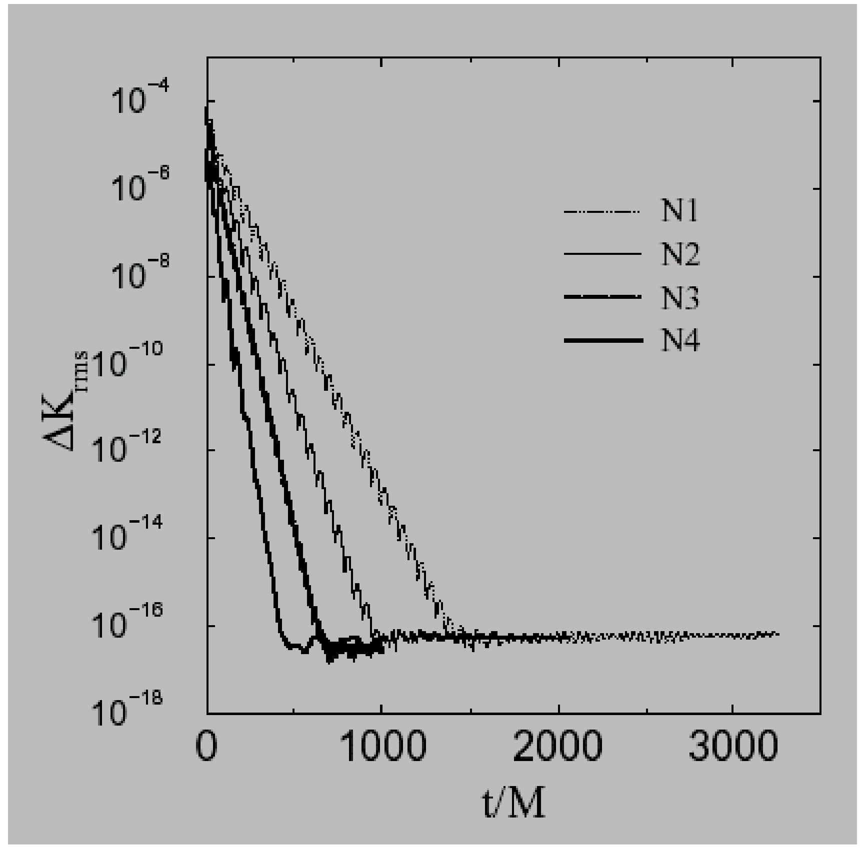
- Lapse function: governs slicing of spacetime
  - **_ = 1 → slices crash into singularity**
  - **"singularity avoiding" slices wrap up around the singularity, but…**
  - **stretching of slices → large gradients**



- Shift vector: governs movement of spatial coords as spacetime evolves
  - **_^{i} = 0 → grid points fall into BH region**
  - **new conditions for shift vector allow shifts that counter slice stretching and enable longer evolutions**
- Application of singularity avoiding slices and new shift conditions → evolve punctures for > 1000 *M* (single BH) and > few x 100 *M* (binary BHs) ***(Alcubierre, et al.)***

Long term evolution of single BHs… *(Yo, et al.)*

- Evolve a single BH, with and without rotation
- Further modification of conformal ADM formulation
- use new lapse and shift from Alcubierre, et al.
- Cubical, spherical excision regions
- Stable for times t > 1000 *M*, for BHs with $J/M \leq 0.9\, M$

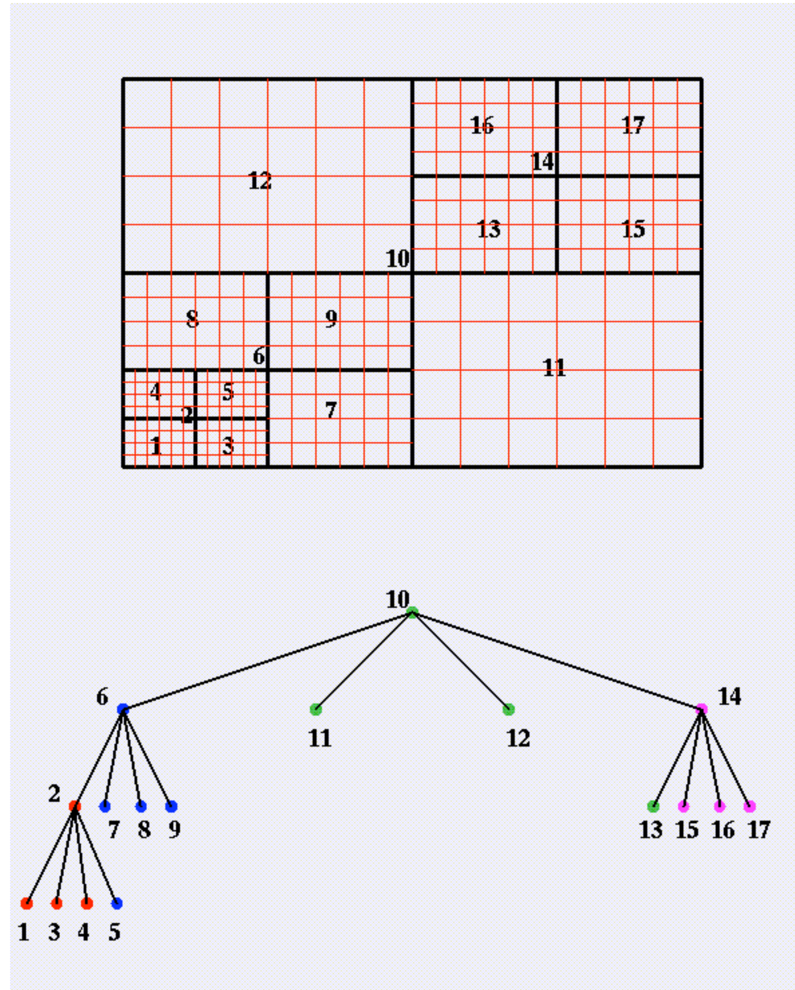


BH mergers…computational challenges

- Strong, dynamical gravitational fields
- Multiple scales:
  - $M_1 \neq M_2$
  - $\_{\mathrm{GW}} \sim 10 - 100 \; L_{\mathrm{source}}$
  - need large enough grid to extract waves, and for outer BCs

→*use of fixed or adaptive mesh refinement (FMR, AMR)*

(This all needs to be done in a parallel computing environment…)

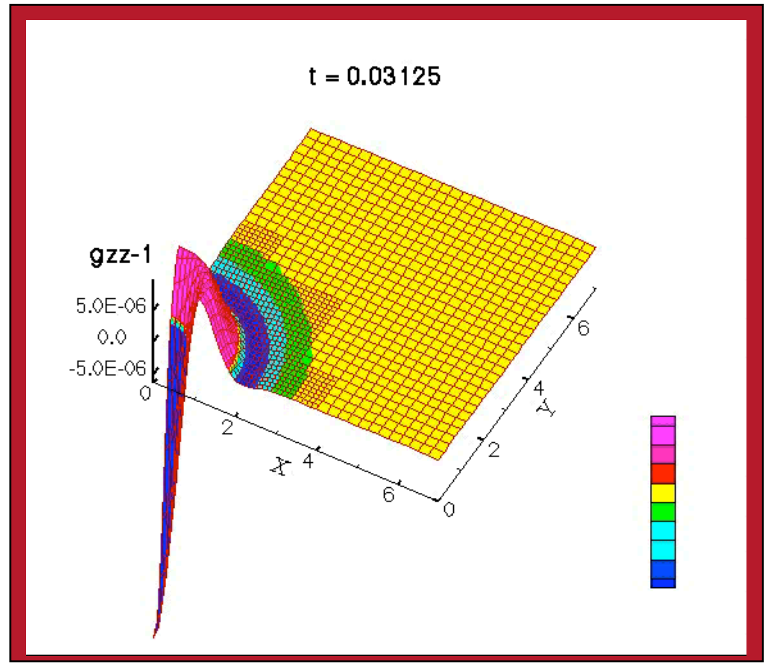


Paramesh (Macneice, et al.)

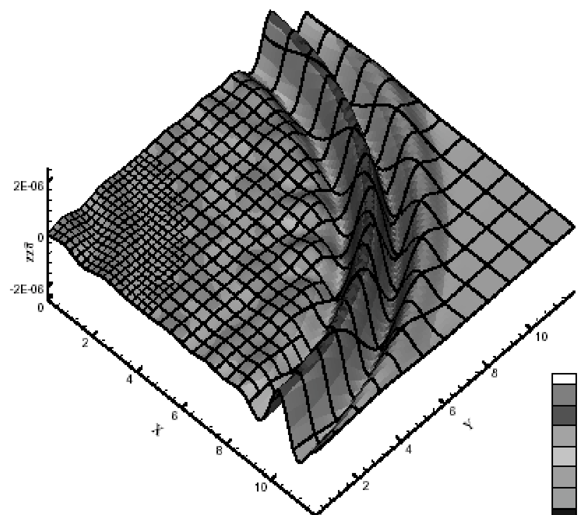
Evolving GWs with mesh refinement...

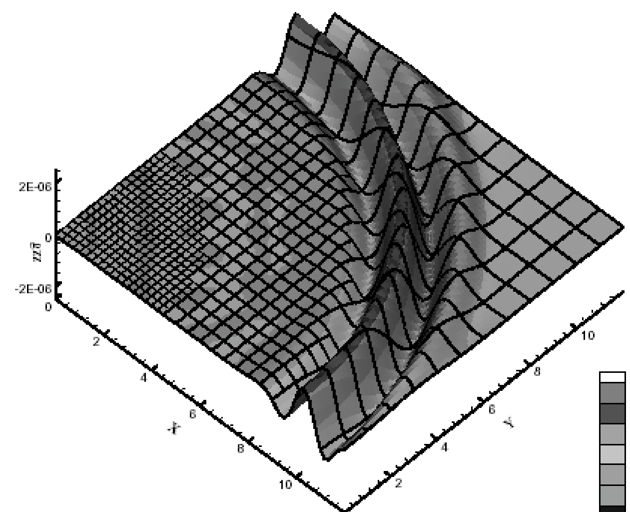
- Evolve source-free GW across grid
- Paramesh for mesh refinement
- With AMR: track the wave →

  (New, et al.)



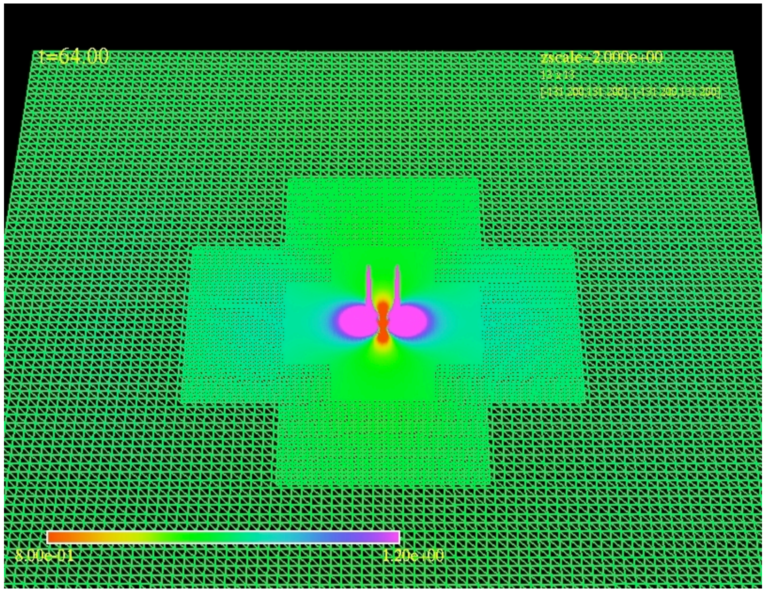
- With fixed mesh refinement:
  - **cross mesh refinement boundaries**
  - **higher order interpolation to remove spurious reflections at interface boundaries *(Choi, et al.)***

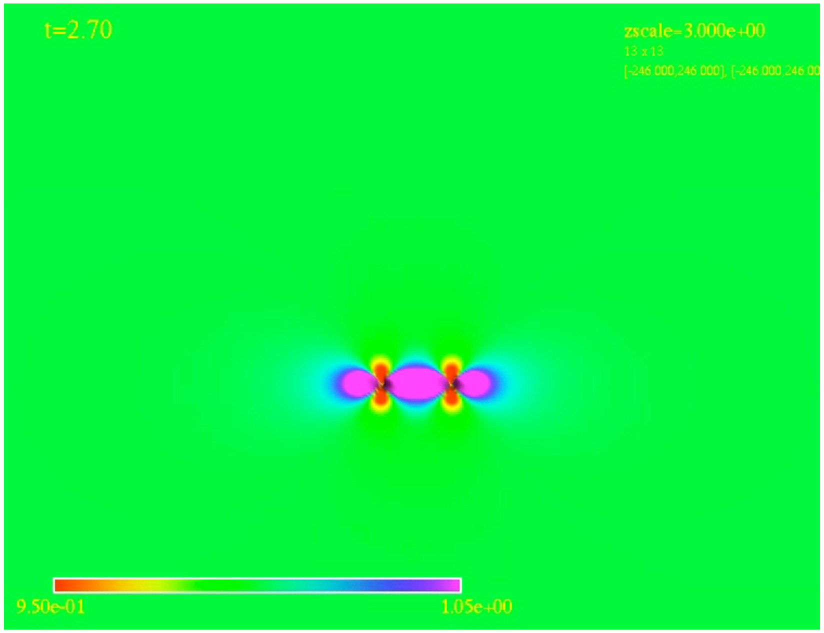


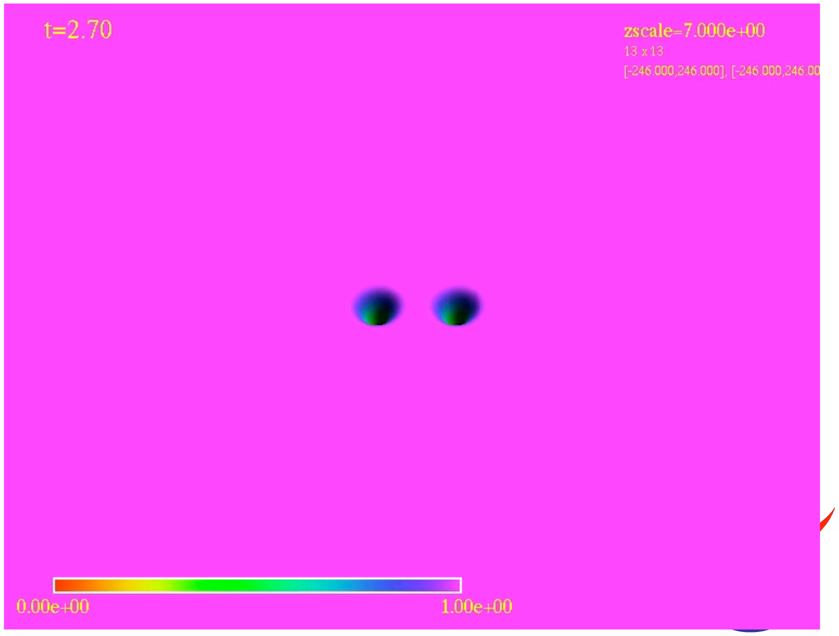


Evolving BHS with FMR…(Choi, et al. 2004)

- **Head-on collision, $M_1 = M_2$**
- **Initial sepn $d = 5M$, $M = M_1 + M_2$**
- **Outer boundary at 120M**
- **5 levels of fixed mesh refinement**
- **Innermost level is located at ~ 8M and has resolution $h \sim M/4$**
- **Metric component g_{xx} and lapse function _ ~ _ (Newtonian potential)**

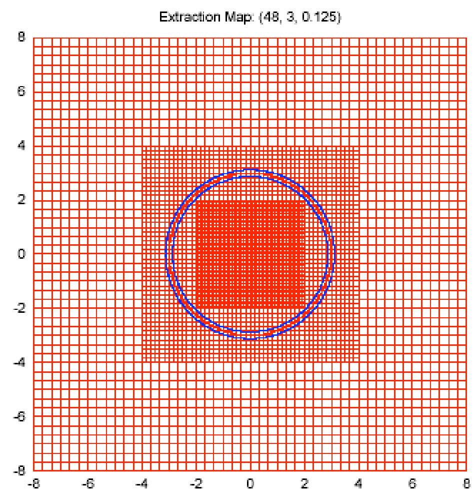


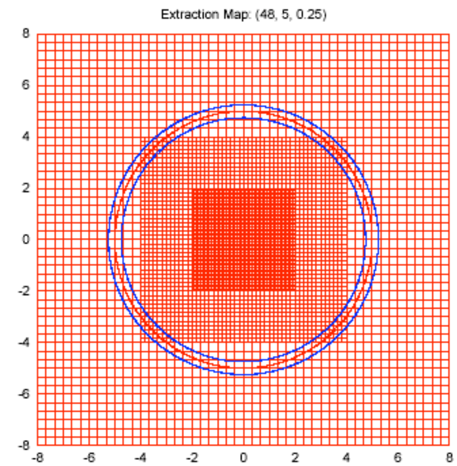


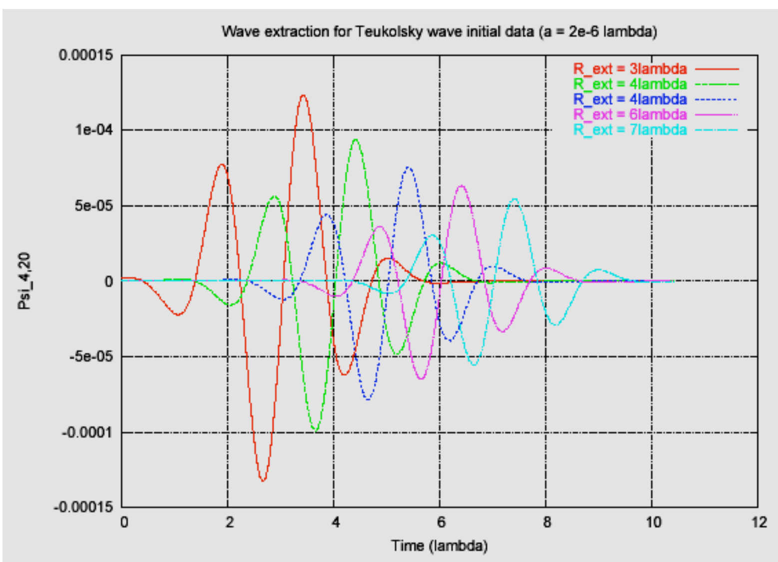


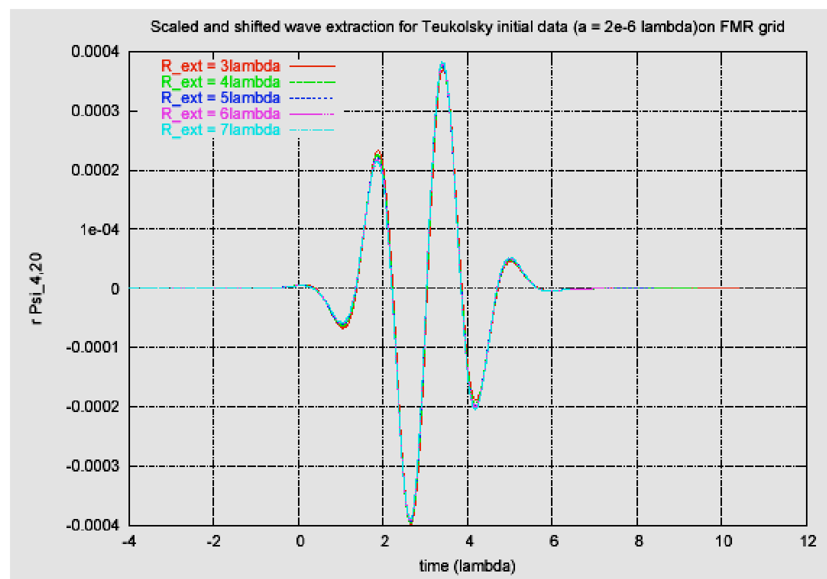
Extracting GWs with FMR…(Fiske et al. 2004)

- Pure GW ($l = 2$, $m = 0$) traversing several FMR regions
- Extract at 5 radii, some crossing FMR boundaries
- Amplitude ~ 1/r
- Scaled waveforms match each other, and analytic solution









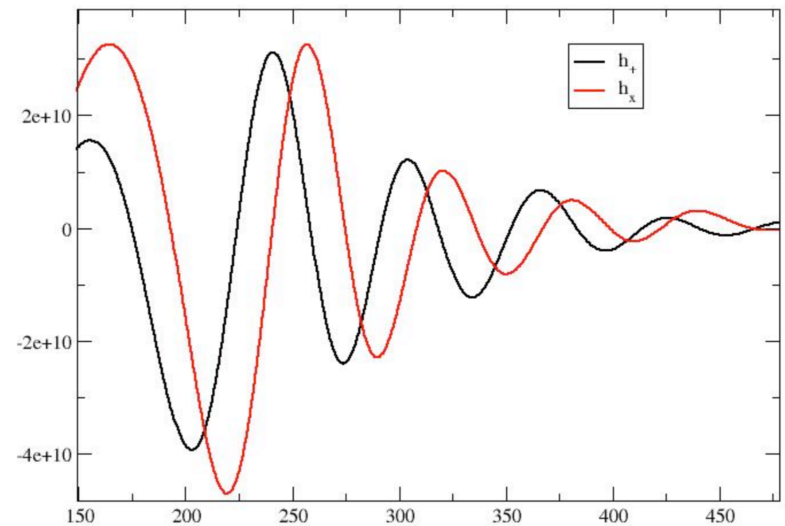
Lazarus: the first astrophysical BH *merger* waveforms . . .

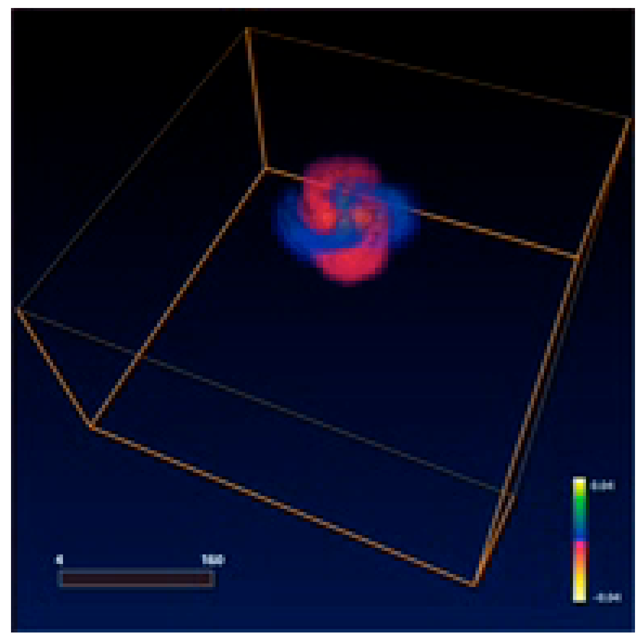
(Baker, Campanelli, Lousto, Takahashi)

- **Coalescence of 2 equal mass nonrotating BHs**
- **Start simulation near last stable circular orbit**
- **Use 3-D Einstein solver (Cactus) to evolve merger until BH perturbation theory becomes applicable**
- **Continue evolving numerically using BH perturbation theory through ringdown → final Kerr BH**
- **~ 3% of total energy emitted as GW**
- **~12% of total J emitted as GW**
- **final Kerr BH has spin ~ $0.7S_{max}$**
- **Time measured in terms of mass:**

→ time in $M = 5 \times 10^{-6}$ sec $(M/M_{\exists})$

graphics courtesy of J. Baker (GSFC)





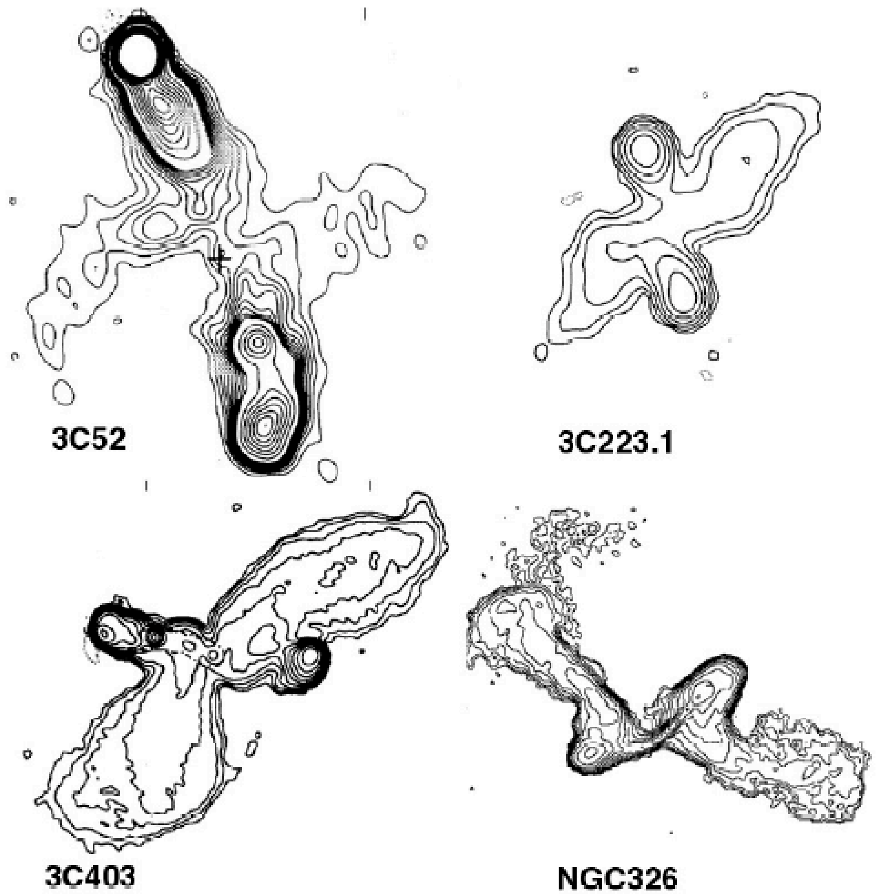
BH binary merger is expected to be phenomenologically rich….

- **Effects of spin: spin-spin and spin-orbit couplings, spin flips…**
- **Mergers of spinning BHs can cause dramatic changes in orientation of BH's spin axis → sudden flip in direction of associated jet**
- **Can identify the winged or X-type radio sources with galaxies in which a merger has occurred (Merritt & Ekers)**
- **Possible ejection of final BH for**

  $M_1 \neq M_2$
- **Tests of General Relativity in the dynamical, strongly nonlinear regime**
- ***Significant challenges remain…***
- ***Numerical relativity has***

  made real progress in modeling binary BH mergers…..

→ stay tuned!



Gravitational Waves . . .

a new kind of cosmic messenger

"Every time you build new tools to see the universe, new universes are discovered. Through the ages, we see the power of penetrating into space."

-- David H. DeVorkin (paraphrasing Sir William Herschel)